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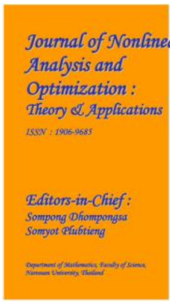

RADIO HARMONIC MEAN LABELING OF CORONA GRAPHS

R. Revathy Department of Mathematics, Shri Shankarlal Sundarbai Shasun Jain College for Women, T.Nagar, Chennai, Tamilnadu. Affiliated to Madras University Email: revathyarunjune2009@gmail.com
Dr. K. Amuthavalli, Department of Mathematics, Government Arts and Science College, Veppanthattai, Perambalur,Tamilnadu, Affiliated to Bharathidasan University Email:thrcka@gmail.com

Abstract:
A radio harmonic mean labeling of a connected graph G is a one to one map f from the vertex set V(G) to the set of natural numbers N such that for two distinct vertices u and v of G, $d(u,v)+\left\lceil \frac{2\ f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G)$ *.The radio harmonic mean number of f,* $rhmn(f)$ *is the maximum number assigned to any vertex of G. The radio harmonic mean number of G,* $rhmn(G)$ *is the minimum value of* $rhmn(f)$ *taken over all radio harmonic mean labeling f of G. In this paper we have determined the radio harmonic mean number of corona graphs..*

Keywords:
Radio harmonic mean labeling, Radio harmonic mean graph, Corona graph
AMS Subject classification: 05C78

1. *Introduction*

All graphs considered here are finite, simple, and undirected. Terms not defined here are used in the sense of Harary [9]. Let V(G) and E(G) denote the vertex set and edge set of the graph G respectively. The channel assignment problems were introduced in 1980 by Hale [8]. The goal is to assign radio channels in a way so as to avoid interference between radio transmitters. Motivated by this Chartrand defined the concept of radio labeling of graphs in 2001[4]. Radio labeling, labels the vertices of a graph with non negative integers such that for any two vertices, the smaller the distance between the vertices, the greater the required difference in label. Radio labeling of graphs is applied in channel assignment problem, Sensor networks, TV and wireless networks etc... S.Ponraj et al. [11,12,13] defined the concept of radio mean labeling of graphs. Motivated by the notion of radio mean labeling we have introduced the radio harmonic mean labeling [1,2,14,15].In this paper, we have investigated the radio harmonic mean labeling of corona graphs and also determined the radio harmonic mean number of those graphs.
In this paper, Radio Harmonic Mean Labeling and Radio Harmonic Mean Number are referred as **RHML** and **RHMN** for briefness.

2. *Definitions*

Definition 2.1
A radio harmonic mean labeling of a connected graph G is one to one map from the vertex set V(G) of G to N such that for two distinct vertices *u* and *v* of G satisfies the condition

$d(u,v)+\left\lceil \frac{2\ f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G).....(1)$ .A graph which admits radio harmonic mean labeling is called radio harmonic mean graph.

Definition 2.2

Radio harmonic mean number of graph G is denoted by $rhmn(G)$. It is defined as the lowest span taken over all radio harmonic mean labeling of graph G.

Definition 2.3

If G is graph of order n, the corona graph of G with another graph H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.

Definition 2.4

Let $u_1, u_2, ..., u_n$ be the vertices of the complete graph K_n. Let $\{v_i, w_i, 1 \leq i \leq n\}$ be the vertices of i^{th} copy of $\overline{K_2}$ which are adjacent to u_i. Then the resultant graph is called $K_n \odot \overline{K_2}$ graph.

Definition 2.5

Let $\{u, u_i : 1 \leq i \leq n\}$ be the vertices of the wheel W_n and let $\{v_i, w_i, 1 \leq i \leq n\}$ be the vertices of $\overline{K_2}$ which are joined to the vertex u_i of the wheel W_n. Then the resultant graph is called $W_n \odot \overline{K_2}$ graph.

3. *MAIN RESULTS*

Theorem 3.1

The radio harmonic mean number of the corona graph $K_n \odot \overline{K}_2$ is $3n+1$ for $n \geq 2$.

Proof

Let the vertex set and edge set be

$$V[K_n \odot \overline{K}_2] = \{u_i, v_i, w_i : 1 \leq i \leq n\} \text{ and}$$

$$E[K_n \odot \overline{K}_2] = \{u_i u_j : 1 \leq i \leq n-1, i+1 \leq j \leq n\} \cup \{u_i v_i, u_i w_i : 1 \leq i \leq n\}.$$

The diameter of $K_n \odot \overline{K}_2$ is 3 for $n \geq 2$.

Define the vertex labels as follows

For $1 \leq i \leq n$,

$$f(u_i) = 2n+1+i$$

$$f(v_i) = 2i$$

$$f(w_i) = 2i+1$$

Now, check the radio harmonic mean condition (1).

In order to verify the definition of radio harmonic mean labeling

$$d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 4$$

for every pair of vertices of $K_n \odot \overline{K}_2$.

Case (1): Verify the pair (u_i, u_j) for $1 \leq i \leq n-1,\ 2 \leq j \leq n,\ j > i,\ d(u_i, u_j) = 1$

$$d(u_i,u_j)+\left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 1+\left\lceil \frac{2(2n+1+i)(2n+1+j)}{4n+i+j+2} \right\rceil \geq 1+\left\lceil \frac{2(6)(7)}{13} \right\rceil \geq 8$$

Case (2): Verify the pair (u_i, v_j) for $1 \leq i, j \leq n,\ d(u_i, v_j) = 1$

$$d(u_i,v_j)+\left\lceil \frac{2f(u_i)f(v_j)}{f(u_i)+f(v_j)} \right\rceil = 1+\left\lceil \frac{2(2n+1+i)(2j)}{2n+i+2j+1} \right\rceil \geq 1+\left\lceil \frac{2(6)(2)}{8} \right\rceil \geq 4$$

Case (3): Verify the pair (u_i, w_j) for $1 \leq i, j \leq n,\ d(u_i, w_j) = 1$

$$d(u_i,w_j)+\left\lceil \frac{2f(u_i)f(w_j)}{f(u_i)+f(w_j)} \right\rceil = 1+\left\lceil \frac{2(2n+1+i)(2j+1)}{2n+i+2j+2} \right\rceil \ge 1+\left\lceil \frac{2(6)(3)}{9} \right\rceil \ge 5$$

Case (4): Verify the pair (v_i,v_j) for $1\le i\le n-1,\ 2\le j\le n,\ j>i,\ d(v_i,v_j)=3$

$$d(v_i,v_j)+\left\lceil \frac{2f(v_i)f(v_j)}{f(v_i)+f(v_j)} \right\rceil = 3+\left\lceil \frac{2(2i)(2j)}{2i+2j} \right\rceil \ge 3+\left\lceil \frac{2(2)(4)}{6} \right\rceil \ge 6$$

Case (5): Verify the pair (v_i,w_j) for $1\le i,j\le n,\ d(v_i,w_j)=2$

$$d(v_i,w_j)+\left\lceil \frac{2f(v_i)f(w_j)}{f(v_i)+f(w_j)} \right\rceil = 2+\left\lceil \frac{2(2i)(2j+1)}{2i+2j+1} \right\rceil \ge 2+\left\lceil \frac{2(2)(3)}{5} \right\rceil \ge 5$$

Case (6): Verify the pair (w_i,w_j) for $1\le i\le n-1,\ 2\le j\le n,\ j>i,\ d(w_i,w_j)=3$

$$d(w_i,w_j)+\left\lceil \frac{2f(w_i)f(w_j)}{f(w_i)+f(w_j)} \right\rceil = 3+\left\lceil \frac{2(2i+1)(2j+1)}{2i+2j+2} \right\rceil \ge 3+\left\lceil \frac{2(3)(5)}{8} \right\rceil \ge 7$$

In all the above cases, f satisfies the radio harmonic mean condition (1).

Thus, f is a radio harmonic mean labeling and $K_n \odot \bar{K}_2$ is a radio harmonic mean graph.

Hence, the radio harmonic mean number of the corona graph $K_n \odot \bar{K}_2$ is $3n+1$ for $n\ge 2$.

Illustration 3.2

RHML of $K_4 \odot \bar{K}_2$ is given below.

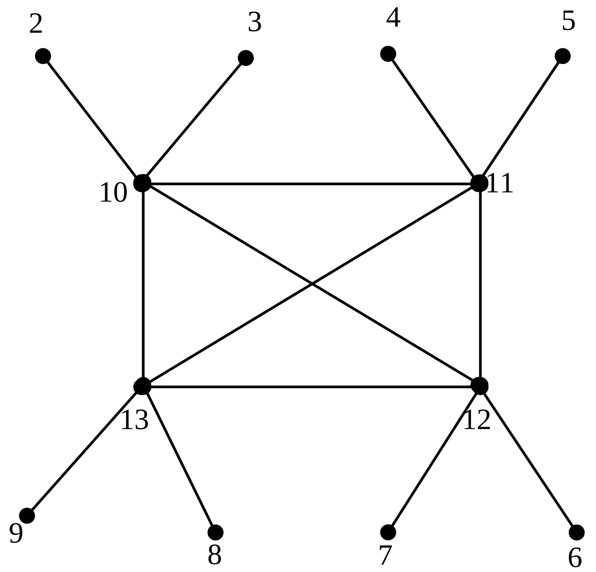

Fig. 3.1: RHML of $K_4 \odot K_2$

Theorem 3.3

The radio harmonic mean number of the corona graph $W_n \odot \bar{K}_2$ is $3n+2$ for $n\ge 3$.

Proof

Let the vertex set and edge set be

$V[W_n \odot \bar{K}_2] = \{u, u_i, v_i, w_i : 1\le i\le n\}$ and

$E[W_n \odot \bar{K}_2] = \{uu_i, u_iv_i, u_iw_i : 1\le i\le n\} \cup \{u_iu_{i+1} : 1\le i\le n-1\} \cup \{u_nu_1\}$.

The diameter of $W_n \odot \bar{K}_2$ is 4 for $n\ge 3$.

Define the vertex labels as follows

$f(u) = 3n+2$

For $1 \le i \le n$,

$f(u_i) = 2n+i+1$

$f(v_i) = i+1$

$f(w_i) = n+1+i$

Now, check the radio harmonic mean condition (1).

In order to verify the definition of radio harmonic mean labeling

$$d(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \ge 5$$

for every pair of vertices of W_n e $\bar{K}_2$.

Case (1): Verify the pair (u,u_i) for $1 \le i \le n$, $d(u,u_i)=1$

$$d(u,u_i) + \left\lceil \frac{2f(u)f(u_i)}{f(u)+f(u_i)} \right\rceil = 1 + \left\lceil \frac{2(3n+2)(2n+i+1)}{5n+i+3} \right\rceil \ge 1 + \left\lceil \frac{2(11)(8)}{19} \right\rceil \ge 11$$

Case (2): Verify the pair (u,v_i) for $1 \le i \le n$, $d(u,v_i)=2$

$$d(u,v_i) + \left\lceil \frac{2f(u)f(v_i)}{f(u)+f(v_i)} \right\rceil = 2 + \left\lceil \frac{2(3n+2)(i+1)}{3n+i+3} \right\rceil \ge 2 + \left\lceil \frac{2(11)(2)}{13} \right\rceil \ge 6$$

Case (3): Verify the pair (u,w_i) for $1 \le i \le n$, $d(u,w_i)=2$

$$d(u,w_i) + \left\lceil \frac{2f(u)f(w_i)}{f(u)+f(w_i)} \right\rceil = 2 + \left\lceil \frac{2(3n+2)(n+1+i)}{4n+i+3} \right\rceil \ge 2 + \left\lceil \frac{2(11)(5)}{16} \right\rceil \ge 9$$

Case (4): Verify the pair (u_i,u_j)

$$d(u_i,u_j) + \left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil$$

Subcase (i): For $1 \le i \le n$, $j=i+1$, $d(u_i,u_j)=1$ and $d(u_n,u_1)=1$

$$d(u_i,u_j) + \left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 1 + \left\lceil \frac{2(2n+1+i)(2n+1+j)}{4n+i+j+2} \right\rceil \ge 1 + \left\lceil \frac{2(8)(9)}{17} \right\rceil \ge 10$$

Subcase (ii): For $1 \le i \le n$, $j > i+1$, $d(u_i,u_j)=2$

$$d(u_i,u_j) + \left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 2 + \left\lceil \frac{2(2n+1+i)(2n+1+j)}{4n+i+j+2} \right\rceil \ge 2 + \left\lceil \frac{2(8)(9)}{17} \right\rceil \ge 11$$

Case (5): Verify the pair (u_i,v_j) for $1 \le i, j \le n$, $d(u_i,v_j)=1$

$$d(u_i,v_j) + \left\lceil \frac{2f(u_i)f(v_j)}{f(u_i)+f(v_j)} \right\rceil = 1 + \left\lceil \frac{2(2n+i+1)(j+1)}{2n+i+j+2} \right\rceil \ge 1 + \left\lceil \frac{2(8)(2)}{10} \right\rceil \ge 5$$

Case (6): Verify the pair (u_i,w_j) for $1 \le i, j \le n$, $d(u_i,w_j)=1$

$$d(u_i,w_j) + \left\lceil \frac{2f(u_i)f(w_j)}{f(u_i)+f(w_j)} \right\rceil = 1 + \left\lceil \frac{2(2n+1+i)(n+1+j)}{3n+i+j+2} \right\rceil \ge 1 + \left\lceil \frac{2(8)(5)}{13} \right\rceil \ge 8$$

Case (7): Verify the pair (v_i,v_j)

$$d(v_i,v_j) + \left\lceil \frac{2f(v_i)f(v_j)}{f(v_i)+f(v_j)} \right\rceil$$

Subcase (i): For $1 \le i \le n,\ j = i+1,\ d(v_i, v_j) = 3$

$$d(v_i,v_j)+\left\lceil \frac{2f(v_i)f(v_j)}{f(v_i)+f(v_j)} \right\rceil = 3+\left\lceil \frac{2(i+1)(j+1)}{i+j+2} \right\rceil \ge 3+\left\lceil \frac{2(2)(3)}{5} \right\rceil \ge 6$$

Subcase (ii): For $1 \le i \le n,\ j > i+1,\ d(v_i, v_j) = 4$

$$d(v_i,v_j)+\left\lceil \frac{2f(v_i)f(v_j)}{f(v_i)+f(v_j)} \right\rceil = 4+\left\lceil \frac{2(i+1)(j+1)}{i+j+2} \right\rceil \ge 4+\left\lceil \frac{2(2)(3)}{5} \right\rceil \ge 7$$

Case (8): Verify the pair (v_i, w_j) for $1 \le i,\ j \le n,\ d(v_i, w_j) = 2$

$$d(v_i,w_j)+\left\lceil \frac{2f(v_i)f(w_j)}{f(v_i)+f(w_j)} \right\rceil = 2+\left\lceil \frac{2(i+1)(n+1+j)}{n+i+j+2} \right\rceil \ge 2+\left\lceil \frac{2(2)(5)}{7} \right\rceil \ge 5$$

Case (9): Verify the pair (w_i, w_j)

$$d(w_i,w_j)+\left\lceil \frac{2f(w_i)f(w_j)}{f(w_i)+f(w_j)} \right\rceil$$

Subcase (i): For $1 \le i \le n,\ j = i+1,\ d(w_i, w_j) = 3$

$$d(w_i,w_j)+\left\lceil \frac{2f(w_i)f(w_j)}{f(w_i)+f(w_j)} \right\rceil = 3+\left\lceil \frac{2(n+1+i)(n+1+j)}{2n+i+j+2} \right\rceil \ge 3+\left\lceil \frac{2(5)(6)}{11} \right\rceil \ge 6$$

Subcase (ii): For $1 \le i \le n,\ j > i+1, d(w_i, w_j) = 4$

$$d(w_i,w_j)+\left\lceil \frac{2f(w_i)f(w_j)}{f(w_i)+f(w_j)} \right\rceil = 4+\left\lceil \frac{2(n+1+i)(n+1+j)}{2n+i+j+2} \right\rceil \ge 4+\left\lceil \frac{2(5)(6)}{11} \right\rceil \ge 7$$

In all the above cases, f satisfies the radio harmonic mean condition (1).

Thus, f is a radio harmonic mean labeling and W_n e $\bar{K}_2$ is a radio harmonic mean graph.

Hence, the radio harmonic mean number of W_n e $\bar{K}_2$ is $3n+2$ for $n \ge 3$.

Illustration 3.4

RHML of W_8 e $\bar{K}_2$ is given below.

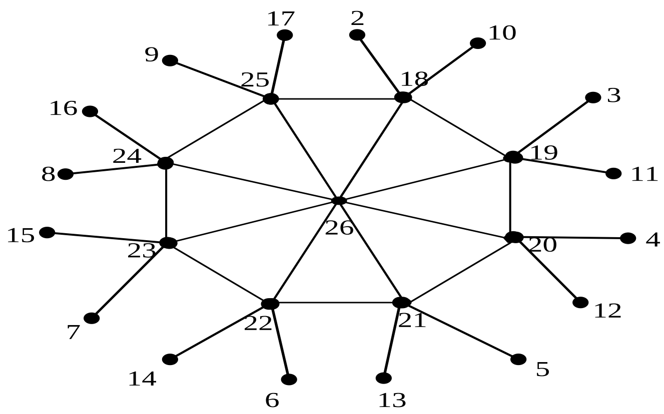

Fig. 3.2: RHML of W_8 e $\bar{K}_2$

4.Conclusion

In this paper, the radio harmonic mean number of corona graphs was obtained. The radio labeling of graphs which would have played a very important role in the communication networks.

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